STRUCTURE PRESERVING REGULARIZATION OF DT-MRI VECTOR FIELDS BY NONLINEAR ANISOTROPIC DIFFUSION FILTERING

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ABSTRACT

In this paper, we propose a PDE-based method for structure preserving regularization of DT-MRI principal diffusion vector fields. We defined a structure sensitive function, the so-called regularity map, which is derived from the local orientation similarity. Regularizing tensor is based on vector field only. Regularization results are found quite satisfactory, in that even simple tracking algorithms accurately reveal the fiber streams in synthetic DTI data.

1. INTRODUCTION

Diffusion Tensor Magnetic Resonance Imaging (DT-MRI) is an image acquisition technique, which is aimed to reflect the structural properties of fibrous tissues, by measuring the diffusion patterns of water molecules. Basically, it fits a 3×3 symmetric, positive semi-definite tensor to each voxel in 3D domain, modeling the water diffusion in every direction. Hence, data acquired by DT-MRI can be considered as a matrix-valued 3D image, which however mostly suffers from noise, as in other MR techniques. The noise, being dependent upon the parameters like acquisition time and voxel size, necessitates regularization/denoising of data, which becomes a crucial step for relaxation of acquisition constraints as well as for applications like fiber tractography, segmentation of brain tissues and measuring brain connectivity.

DT-MRI based in-vivo fiber tractography (FiT) is based on following the principal eigenvectors of diffusion tensors, aiming to track the fibers, which are usually supposed to exhibit a regular and anisotropic flow structure. For distinguishing the segments of these fiber bundles, Fractional Anisotropy (FA) can be used as a conventional scalar metric that reflects the dominance of diffusion tensor's larger eigenvalue over the other ones.

As in [2], traditional PDE based approaches can be exploited to regularize the scalar image of FA, for restoring DT-MRI data. In fact, PDE methods have been widely used through different applications and formulations for solving the problem of scalar image denoising since the pioneering work of Perona and Malik [1]. In this study, we use their extension to the Principal Diffusion Direction (PDD) vector field.

There are recently derived PDE-schemes, addressed to denoising of vector valued fields including color image restoration [4], which take possible coupling between vector components into account, while performing the regularization process on each of them separately. In the context of DT-MRI Tschumperlé and Deriche adopted a simple anisotropic 2D/3D PDE-method acting on the coefficients of the diffusion tensor, while adding constraints in terms of symmetry and semi-positivity [5]. They also attempted to regularize diffusion tensor's spectral components in terms of orientations (eigenvectors) and diffusivities (eigenvalues), by proposing a new orthogonal matrix diffusion technique. Apart from the use of PDEs, Coulon [2] also used a variational approach to restore PDD fields, where they aimed to minimize a total variation based energy defined for the direction map. On the other hand, topology preserved smoothing of vector fields is addressed in [3], where vector data is first converted into a scalar representation, treating time surfaces as level-sets. By analyzing the dynamic behavior of these level sets, they determine distinct flow features, and, vector fields are successively smoothed by combining geometrical and topological considerations, while keeping their flow structure unchanged. In most of these studies, especially those which deal with PDD fields, the directional indeterminancy of unit vectors, i.e. the equivalence condition of antipodal directions is locally provided, for instance by an initial eigenvector alignment [5]. However this may introduce forged discontinuities or remove the actual ones.

In this paper, we employ anisotropic PDE-methods, for efficiently regularizing PDD fields, particularly within and along fiber tracts only. As a novel attempt, we let them act on the elements of the orientation tensor, a 3×3 symmetric positive semi-definite matrix, which is defined as the tensor product of PDD vectors solving the above mentioned antipodal direction ambiguity. We propose a new robust measure called the regularity map as the structure sensitive map, which is also derived from the orientation tensor field. Coupled with the PDD field itself, the regularity map, which accurately reflects the discontinuities between fiber and non-fiber regions, allows us to design a nonlinear anisotropic diffusion system. It can adapt itself to the local structure and selectively smooth the vector flow along fiber bundles. Another contribution of the proposed method is that the smoothing tensor used in filtering is generated based on the vector field only. No additional information such as FA map is used. Thus the proposed method is applicable to other types of vector fields as well.

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The paper is organized as follows: In section 2, we discuss our structure preserving smoothing objectives. Section 3 and 4, propose the vector orientation field as the matrix-valued image to be regularized, and regularity map *f* as the structure sensitive function, respectively. Section 5 defines our regularizing diffusion process. Finally, in section 6 we give regularization and tracking results on synthetic data and in section 7 we make our conclusions.

2. THE DESIGN OBJECTIVES

PDD vector field of DT-MR images, consists of 3D unit vectors (principal eigenvector of diffusion tensors). In Figure 1. two physiological segments are synthetically distinguished as regions shaded with gray corresponding to a non-fiber and white for fiber. Note the difference in the regularity of the PDD in both regions, where the fiber wall is defined as the interface inbetween, i.e. the discontinuity to be preserved. Briefly, in order to achieve accurate tracts from FiT algorithms, we simply aim to improve the regular flow within and along those tracts while keeping the randomness in nonfiber areas, retaining and preferably enhancing their boundary. In other words PDEs to be employed have to accomplish the following diffusion patterns:

- Isotropic and strong smoothing inside fiber region
- Isotropic and weak smoothing inside non-fiber region
- Anisotropic smoothing along fiber wall

3. THE VECTOR ORIENTATION FIELD

We considered the so called vector orientation field **V**, instead of PDD vectors **v** themselves, as the basis of regularizing diffusion because the directional differences, especially those of neighboring antipodal vectors introduce large synthetic discontinuities. The orientation field **V** is simply computed by taking the tensor product of PDD vectors, which maps them from R^3 to R^6 (symmetry allows us

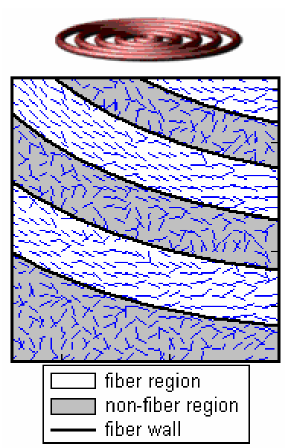

Figure 1: Top: Geometry of ring phantom used in this study, Bottom: Its zoomed PDD field (top view) with indicated regions of concern

to take the upper triangular 6 coefficients of the resulting orientation tensor), preserving distances but assigning antipodal ones to the same term:

$$\mathbf{V} = \mathbf{v}\mathbf{v}^T$$

Clearly, **V**'s principal eigenvector is still **v** with the corresponding largest eigenvalue 1, and 0 for the others. From now on, as in [5], its coefficients can be treated as separate channels of a multi-valued image, during the regularization process.

4. THE REGULARITY MAP

An immediate candidate for the structure sensitive function that distinguishes between fiber and non fiber tissues, would be the FA map which is widely used with DT-MRI data. However as it can be seen for the phantom rings in Figure 2 (left), it can exhibit noisy characteristics inherited from diffusion tensors. FA also discards the knowledge of dominant flow direction. A better choice would be a regularity map, *f*, which utilizes the similarity within a given neighborhood of vectors in terms of their orientations. *f* is defined as

$$f = FA\{\mathbf{V}_\sigma\} = \frac{\sqrt{3}}{\sqrt{2}} \frac{\sqrt{(\lambda_1-\bar{\lambda})^2+(\lambda_2-\bar{\lambda})^2+(\lambda_3-\bar{\lambda})^2}}{\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}$$

where $\mathbf{V}_\sigma = \mathrm{K}(\sigma)*\mathbf{V}$ is a smoothed version of **V** convolved with a spherical 3D Gaussian kernel $\mathrm{K}(\sigma)$ having a standard deviation σ, which can be selected according to the resolution of data. $\lambda_1 \geq \lambda_2 \geq \lambda_3$ are the magnitude ordered eigenvalues of $\mathbf{V}_\sigma$, with their average $\bar{\lambda}$. Smoothing of **V** with the Gaussian kernel, makes its second and third eigenvalues nonzero to the extent of the local PDD irregularity. For locally coherent regions, typically for fiber interiors, the diffusivity contained in these second and third eigenvalues will still be small, resulting in a high anisotropy, i.e. a large *f*, close to 1. On the other hand, non-fiber regions, where vectors within a neighborhood exhibit a rather random structure, result in a spherical distribution of diffusivity over the three eigenvalues of $\mathbf{V}_\sigma$ and concordantly in a low *f*, close to 0. As it can be seen from Figure 2 (right), *f* reflects the desired structure, especially at fiber walls, as critical discontinuities to be preserved.

5. THE REGULARIZING DIFFUSION

The anisotropic diffusion PDE, that we separately apply on the 6 upper triangular coefficients of the orientation tensor **V** is formulated as:

$$\frac{\partial V_i}{\partial t} = \nabla \bullet \left(\mathbf{D}\nabla V_i\right) \qquad (i = 1 \ldots 6)$$

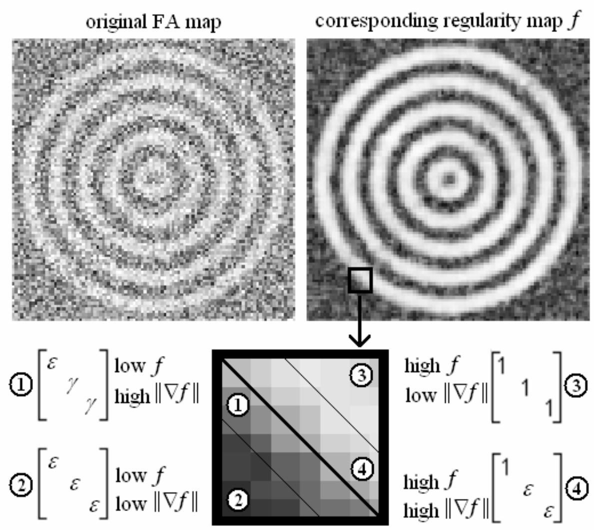

Figure 2: Top-Left: Noisy FA map; Top-Right: Regularity map *f* from the same noisy field, Bottom: An enlarged frame of discontinuity, with corresponding structure sensitive measures *f* and $\|\nabla f\|$, and the eigenvalues of the smoothing tensor ($0 < \gamma << \varepsilon << 1$).

where **D** is the 3×3 symmetric, positive semi-definite, smoothing tensor to be determined according to the desired regularization characteristics. **D** drives the regularizing diffusion process by enhancing or suppressing the amounts of smoothing selectively along its eigendirections. **D** is defined as:

$$\mathbf{D} = [\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3] \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix} \begin{bmatrix} \mathbf{e}_1^T \\ \mathbf{e}_2^T \\ \mathbf{e}_3^T \end{bmatrix}$$

μ_i's determine the diffusion (i.e. the extent of smoothing) along $\mathbf{e}_i$. Setting $\mathbf{e}_i$'s and μ_i's is the crucial point of the proposed algorithm. We would like to have (i) the dominant diffusion to be along the fiber orientation around the fiber wall (ii) isotropic diffusion away from the fiber wall (iii) the average diffusion to be higher inside the fiber than outside.
We defined $\mathbf{e}_i$'s, to meet the above goals, as follows:

$$\mathbf{e}_1 = \frac{\mathbf{v} - \left(\mathbf{v} \bullet \frac{\nabla f}{\|\nabla f\|}\right)\frac{\nabla f}{\|\nabla f\|}}{\left\|\mathbf{v} - \left(\mathbf{v} \bullet \frac{\nabla f}{\|\nabla f\|}\right)\frac{\nabla f}{\|\nabla f\|}\right\|}, \quad \mathbf{e}_2 = \frac{\nabla f}{\|\nabla f\|}, \quad \mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2$$

$\mathbf{e}_1$ is tangent to the fiber wall and is close to the local DTI vector $\mathbf{v}$, $\mathbf{e}_2$ is perpendicular to the fiber wall and $\mathbf{e}_3$ is defined to be orthogonal to $\mathbf{e}_1$ and $\mathbf{e}_2$. Corresponding eigenvalues of **D**, giving the weighted amounts of directional smoothing, are given as:

$$\mu_1 = g(f), \quad \mu_2 = g(f)h(\|\nabla f\|), \quad \mu_3 = \mu_2 = g(f)h(\|\nabla f\|)$$

where the diffusivities *g* and *h* are defined as:

$$g(f) = \exp\left\{-C(f-1)^{2\kappa}\right\}, \quad h(\|\nabla f\|) = 1 - \exp\left\{\frac{-D}{\left(\frac{\|\nabla f\|}{\rho}\right)^{\eta}}\right\}$$

with their parameters C, κ, D, ρ and η having been determined empirically. Note that both *g* and *h* with a common range of [0,1], are monotone sigmoid functions of their arguments. Thus, for fiber interiors, with typically large *f* and small $\|\nabla f\|$, all weights become close to 1, resulting in a large isotropic smoothing, while for either side of the fiber wall only the first weight, i.e. the one which acts parallel to the PDD flow will be large, and in fact proportional to the local regularity. For detailed distribution of those weights see Figure 2, (bottom), where the corresponding eigenvalues of **D** are indicated as diagonal elements for each of 4 different types of local structure.

6. EXPERIMENTAL RESULTS

In our experiments, with typical diffusion values, high resolution mathematical ring phantoms (with FA_{fiber}=0.82, $FA_{nonfiber}$=0.13) are used, where all fiber diameters were chosen to be 1 unit [7]. Data contains Gaussian noise added to real and imaginary parts of each complex MR signal where signal to noise ratio (SNR) is defined as the reciprocal of noise standard deviation. Figure 1 shows the geometry of ring phantoms, and the associated noisy PDD field with SNR=8. We iteratively smoothed each of 6 scalar upper triangular channels of the vector orientation field **V**, using the above defined diffusion process with a time step of Δt=0.25 and until the weighted regularity *f* in fiber regions reaches a certain threshold τ. Weighting is done by masking *f* with its initial nonlinear map *g*(*f*). Restored field is generated by taking the principal eigenvector of **V**. Figure 4 shows the regularization results with the optimum set of parameters, which are empirically set to $\sigma = 0.5$, $C = 10^7$, $\kappa = 6$, $D = 4$, $\rho = 0.06$, $\eta = 8$ and $\tau = 0.95$. We also examined the performance of the proposed method by tracking the fibers, comparatively in both original and restored fields. Figure 4 shows fibers that are initiated at identical seed points, which are randomly selected within fibers. PDDs are linearly interpolated at successive track points. Extracted tracts consist of at most 1000 vertices, spaced with a step size of 0.5 × voxel-width. For the original PDD field, conventional termination criterion is used, which is FA falling below 0.15 with curvature < 20^o (Figure 4 (top)). For the restored field we used the regularity map *f* falling below 0.80 and again with curvature < 20^o (Figure 4 (bottom)).

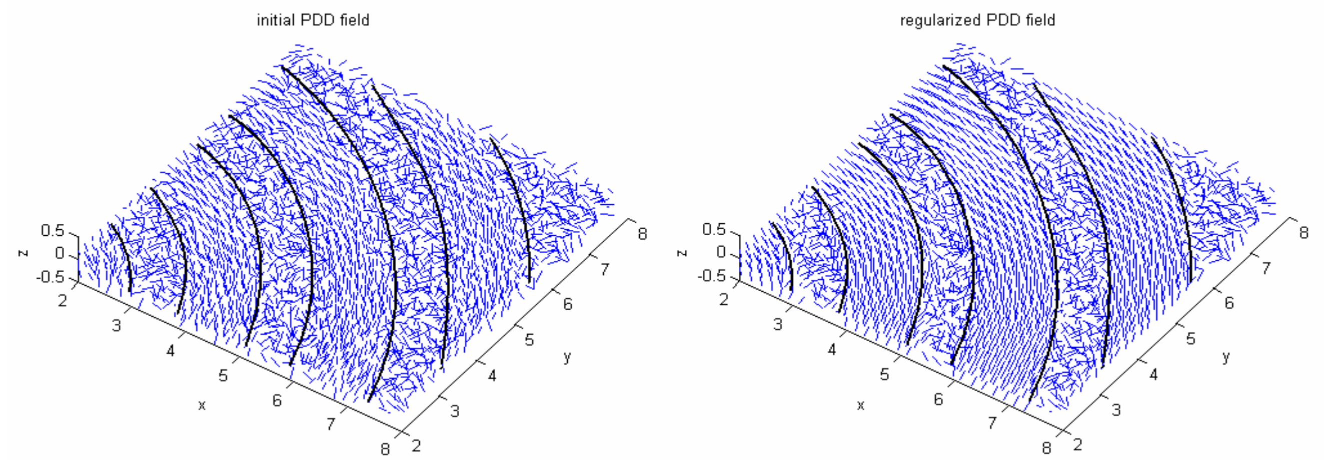

Figure 3: A 30×30×3 subvolume of original and regularized PDD fields with their fiber walls indicated. Signal to noise ratio (SNR)=8, Sampling interval (fs)=0.2mm, $\sigma = 0.5$, $C = 10^7$, $\kappa = 6$, $D = 4$, $\rho = 0.06$, $\eta = 8$ and $\tau = 0.95$.

7. CONCLUSIONS

The method is sensitive to the structure in a vector field. It can differentiate regular and irregular regions without using additional information, thus it is applicable to other vector fields than the DTI data. Simulation results in phantom data with even low SNR values are very promising, in that the proposed PDE-based smoothing scheme rapidly converges to highly regular vector fields for fiber interiors, while preserving discontinuities and keeping the non-fiber regions almost unchanged. No leakage of region characteristics is observed. Using *f* as the termination criteria of the tracking process, allows us to prevent the tracts diverge from their actual routes. We evaluated the performance of the proposed regularization scheme visually. In our future studies quantitative performance criteria will also be provided. The method requires to be examined in other phantom geometries and real patient data as well.

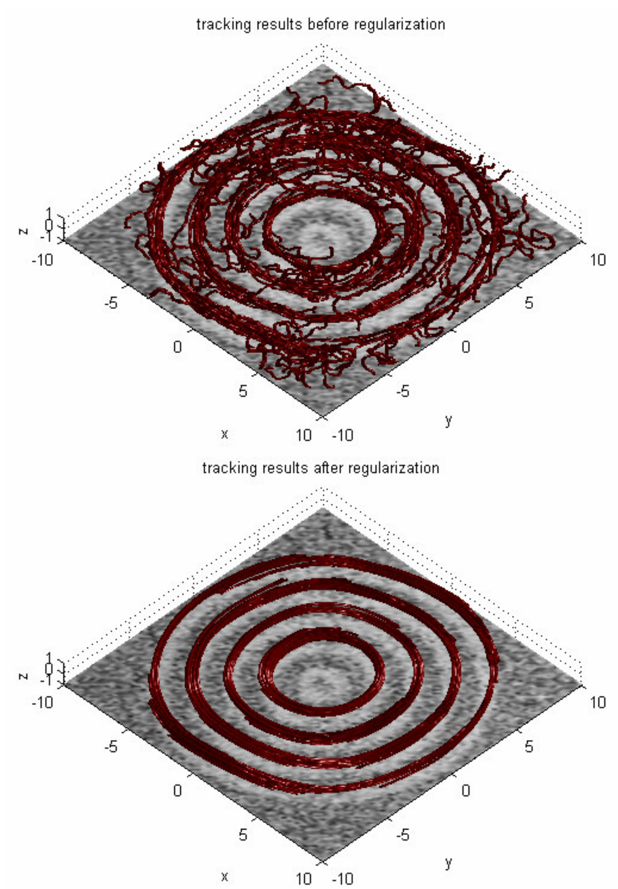

Figure 4: Tracking results before and after regularization of PDD field

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