Generalized Diffusion Tensor Imaging (GDTI) Using Higher Order Tensor (HOT) Statistics

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ABSTRACT

Diffusion tensor imaging (DTI) is known to have limited capability of resolving multiple fiber orientations within one voxel. Angular distribution of the apparent diffusion coefficient is incapable of inferring the correct fiber orientations as demonstrated in this abstract. A new methodology is proposed by generalizing the Fick's law to a higher order partial differential equation and reconstructing the probability density function of the displacement using higher order cumulants. The higher order cumulants can be measured using conventional diffusion-weighted imaging techniques. Simulations demonstrate that this method is capable of accurately resolving multiple fiber orientations.

THEORY

Fick's first law can be generalized as,

$$F_i = -D^{(2)}_{ij}\frac{\partial C}{\partial x_j} - D^{(3)}_{ijk}\frac{\partial^2 C}{\partial x_j \partial x_k} - D^{(4)}_{ijkl}\frac{\partial^3 C}{\partial x_j \partial x_k \partial x_l} - \cdots \qquad [1]$$

Here the coefficient $D^{(n)}_{i_1 i_2 \cdots i_n}$ is an *n*-th order diffusion tensor, *F* is the flux and *C* is concentration. Using Eq. [1], and following Torrey's derivation (1), the transverse magnetization is found to be,

$$m\left(b^{(n)}\right) = m(0)\exp\left(\sum_{n=2}^{\infty}(+j)^n D^{(n)}_{i_1 i_2 \cdots i_n} b^{(n)}_{i_1 i_2 \cdots i_n}\right) \qquad [2]$$

where $m(0)$ is the transverse magnetization in the absence of diffusion gradient and $b^{(n)}$ is an *n*-th order tensor determined by diffusion gradients. Useful information is contained in both the magnitude and phase of the signal. The higher order tensors (HOT) $D^{(n)}_{i_1 i_2 \cdots i_n}$ can be estimated by combining a series of diffusion-weighted images. The higher order cumulants can then be calculated using these tensors. The probability density function (PDF) of the displacement can be reconstructed based on those cumulants by using Gram-Charlier series. Generally this PDF is non-Gaussian as apposed to the Gaussian PDF computed by conventional DTI.

METHOD

Simulations were done on four synthetic phantoms: (i) an isotropic phantom, (ii) a single tube, (iii) two perpendicularly crossing tubes, and (iv) a Y-shaped tube. The following imaging sequence parameters were used: the duration of the diffusion gradient δ = 20.2ms, their separation Δ = 100.5ms and D = 2.02×10^{-3} mm^2/s. 200 diffusion gradient directions evenly distributed on the surface of a sphere were applied. At each orientation the gradient strength was varied from 0 – 40 mT/m in 10 uniform steps. The generalized diffusion tensors up to order 4 were estimated using the SVD method. The PDF of spin displacement was reconstructed by calculating its higher order cumulants. PDF iso-surface plot and skewness plot were used to visualize the reconstructed fiber structure. In the skewness map, green represents positive values and red represents negative values. Hence, green indicates the area where the probability density increases.

RESULTS

The resultant PDF iso-surface plots and the skewness maps obtained from the four phantoms are plotted in Fig. 1. In a side-by-side comparison, the diffusion ellipsoid determined by conventional DTI and the angular distribution of ADC determined by high angular resolution diffusion weighted imaging (HARD) method (2) are also shown in this figure. The skewness map is plotted in two ways: in the first plot, the negative part (red) is made transparent while the positive part (green) is solid; in the other plot, both parts are made solid. All those plots are 3D objects. The HOT method revealed the structure of the phantoms accurately as seen in the PDF plots. Neither DTI nor HARD can identify the crossing and Y-shaped structure correctly.

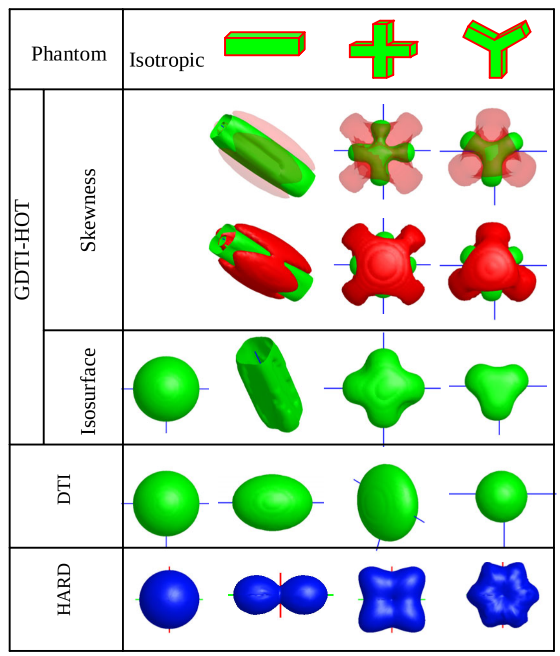

Fig. 1 – Comparison of HOT, DTI and HARD. For isotropic phantom, the skewness is too small to be significant. Hence they are not plotted here. In the PDF and diffusion ellipsoid plots, the blue lines are x and y axis; in the HARD image, red line is the y axis and green is the x axis.

DISCUSSION

In brain tissues it is, to some extent, likely that, with the current spatial resolution of MRI, more than one population of fiber tracts can be situated within one voxel. As demonstrated by the simulations, the HOT method can accurately reveal the underlying structure for all four phantoms, whereas DTI and HARD fail to identify the simulated intersecting fibers. Both DTI and HARD only utilize the first term on the left hand side of Eq. [1], hence will suffer from the unrecoverable information loss. HOT also has the potential to greatly improve tractography (3).

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REFERENCES

1. Torrey HC, Phys Rev 1956; 104: 563-565.
2. Tuch DS, et al. Proceedings of the 7th Annual Meeting of ISMRM, 1999, p321.
3. Basser PJ, et al. Magn Reson Med 2000; 44: 625-632.